المحاضرة 12

$$b_K = \frac{1}{\pi}\left[\int_{-\pi}^{0} -x \sin Kx \, dx + \int_{0}^{\pi} x \cdot \sin Kx \, dx\right]$$

$$= \frac{1}{\pi}\left[-\left(\frac{-x\cos Kx}{K}\Big|_{-\pi}^{0} + \int_{-\pi}^{0} \frac{\cos Kx}{K}\,dx\right) + \left(\frac{-x\cos Kx}{K}\Big|_{0}^{\pi} + \int_0^{\pi} \frac{\cos Kx}{K}\right)\right.$$

$$= \frac{1}{\pi}\left(-0 + \pi\frac{\cos K\pi}{K} - \frac{1}{K^2}\sin Kx\Big|_{-\pi}^{0} - \pi\frac{\cos K\pi}{K} + 0 + \frac{1}{K^2}\sin Kx\Big|_{0}^{\pi}\right)$$

$$= \frac{1}{\pi K^2}\left(-0 - \sin \pi K + \sin \pi K - 0\right)$$

نشر فورييه $f(x) = \frac{\pi}{2} + \frac{-4}{\pi(1)^2}\cos x + \frac{-4}{\pi(3)^2}\cos 3x + \ldots$

أوجد نشر فورييه للدالة الدورية لـ $f(x)$ التي دورها (2π) و المعرفة بالشكل

$$f(x) = \begin{cases} 0 & -\pi \le x < 0 \\ x^2 & 0 \le x < \pi \end{cases}$$

$$T_0 = 2\pi \Rightarrow w_0 = \frac{2\pi}{T_0} = 1$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{0} f(x)\,dx + \int_0^{\pi} x^2 \, dx$$

$$a_0 = \frac{1}{\pi}\,\frac{x^3}{3}\Big|_0^{\pi} = \frac{1}{\pi}\left(\frac{\pi^3}{3} - 0\right) = \frac{\pi^3}{3}$$

$$a_K = \frac{1}{\pi}\int_{0}^{\pi} \underbrace{x^2}_{u} \underbrace{\cos Kx \, dx}_{dv} = \frac{1}{\pi}$$

$dv = \cos kx \Rightarrow v = \frac{\sin kx}{k}$

$u = x^2 \Rightarrow du = 2x\, dx$

$$= \frac{1}{\pi}\left(x^2 \cdot \frac{\sin kx}{k}\Big|_0^{\pi} - 2\int_0^{\pi} x\, \frac{\sin kx}{k}\, dx\right)$$

تجزئة ثانية:

$$\cancel{a_k = \frac{1}{\pi}\left(\pi^2 \frac{\sin k\pi}{k}\Big|_0^{\pi} - 2\int_0^{\pi} x\, \frac{\sin kx}{k}\, dx\right)}$$

$$a_k = \frac{1}{\pi}\left(\pi^2 \frac{\sin k\pi}{k} - 0 - \frac{2}{k}\left(-x\, \frac{\cos kx}{k}\Big|_0^{\pi} + \int_0^{\pi} \frac{\cos kx}{k}\, dx\right)\right)$$

$$= \frac{1}{\pi}\left(0 - 0 - \frac{2}{k}\left(-\pi\, \frac{\cos k\pi}{k} + 0 + \frac{1}{k}\left(\frac{\sin kx}{k}\Big|_0^{\pi}\right)\right)\right)$$

$$= \frac{1}{\pi}\left(\frac{2\pi}{k^2}\cos k\pi + 0\right) = \frac{2}{k^2}\cos k\pi = \begin{cases} \frac{2}{k^2} & k \text{ زوجي} \\ -\frac{2}{k^2} & k \text{ فردي} \end{cases}$$

$$b_k = \frac{1}{\pi}\int_0^{\pi}(x^2 \cdot \sin kx\, dx)$$

$$= \frac{1}{\pi}\left(-x^2 \cdot \frac{\cos kx}{k}\Big|_0^{\pi} + 2\int_0^{\pi} \frac{\cos kx}{k} \cdot x\, dx\right)$$

$$= \frac{1}{\pi}\left(\left(-\pi^2 \cdot \frac{\cos k\pi}{k} - 0 + \frac{2}{k}\left(\left(x \cdot \frac{\sin kx}{k}\Big|_0^{\pi} - \int_0^{\pi} \frac{\sin kx}{k}\, dx\right)\right)\right)\right)$$

$$= \frac{1}{\pi k}\left(-\pi^2 \cdot \cos k\pi + 2\left(\frac{\pi \sin k\pi}{k} - 0\right) + \frac{2\cos kx}{k^2}\Big|_0^{\pi}\right)$$

$$= \frac{1}{K\pi}\left(\frac{-\pi^2 \cdot \cos K\pi}{\frac{\pi K^2}{K^2}} + \frac{2}{K^2}(\cos K\pi - 1)\right)$$

$$= \frac{1}{\pi K^3}\left((-\pi^2 K^2 + 2)\underbrace{\cos K\pi}_{\cos 2\pi = 1} - 2\right) = \begin{cases} K \text{ زوجي} \\ K \text{ فردي} \end{cases}$$

زوجي $K = 2 \Rightarrow \cos 2\pi = 1 \Rightarrow \frac{-\pi^2 K^2 + 2 - 2}{\pi K^3} = -\frac{\pi}{K}$ زوجي

فردي $K = 1 \Rightarrow \cos 1\pi = -1 \Rightarrow -\pi^2 K^2 - 2 - 2$ $\quad -4$

$$\Rightarrow \frac{\pi^2 K^2 - 4}{\pi K^3}$$ فردي

نشر فورييه

$$f(x) = \frac{\pi^2}{6} + \left(\frac{-2}{K^2}\cos x + \frac{\pi^2(1)^2 - 4}{\pi(1)^2}\sin x + \frac{2}{(2)^2}\cos 2x + \left(\frac{-\pi}{2}\sin 2x\right)\right.$$

* اوجد نشر فورييه للدالة الدورية $f(x)$ التي دورها (2π) ، المعرفة بالشكل

$$f(x) = \begin{cases} +x & x > 0 \\ -x & x < 0 \end{cases}$$

$$T_0 = \pi \Rightarrow w_0 = \frac{2\pi}{T_0} = \frac{2\pi}{\pi} = 2$$

$$\frac{\pi^2}{8} + \frac{\pi^2}{8} = \frac{2\pi^2}{8}$$

$$a_0 = \frac{2}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (|x|\,dx) = \frac{2}{\pi}\left(\int_{-\frac{\pi}{2}}^{0} -x\,dx + \int_{0}^{\frac{\pi}{2}} x\,dx\right)$$

$$\Rightarrow a_0 = \frac{2}{\pi}\left(\frac{-x^2}{2}\Big|_{-\frac{\pi}{2}}^{0} + \frac{x^2}{2}\Big|_{0}^{\frac{\pi}{2}}\right) = \frac{2}{\pi}\left(0 + \frac{\frac{\pi^2}{4}}{2} + \frac{\frac{\pi^2}{4}}{2} - 0\right) = \frac{2}{\pi}\left(\frac{\pi^2}{4}\right) = \frac{\pi}{2}$$

$$a_K = \frac{2}{\pi}\left[\int_{-\frac{\pi}{2}}^{0} \underbrace{-x}_{u} \cdot \underbrace{\cos(2Kx)\,dx}_{dv} + \int_{0}^{\frac{\pi}{2}} \underbrace{x}_{u} \cdot \underbrace{\cos(2Kx)\,dx}_{dv}\right]$$

$$= \frac{2}{\pi}\left[-\left(x \cdot \frac{\sin(2kx)}{2k} \Big|_{-\frac{\pi}{2}}^{0} - \int_{-\frac{\pi}{2}}^{0} \frac{\sin(2kx)}{2k}\, dx \right) + \left(x \cdot \frac{\sin(2kx)}{2k} \Big|_{0}^{\frac{\pi}{2}} + \int_{0}^{\frac{\pi}{2}} \frac{\sin 2kx}{2k}\, dx \right) \right.$$

~~$u = x^2$ du~~

$$= \frac{2}{\pi}\left[-\left(0 - \frac{\pi}{2}\,\frac{\sin k\pi}{2k}\right) - \frac{1}{2k}\,\frac{\cos 2kx}{2k}\Big|_{-\frac{\pi}{2}}^{0} + \right.$$

نعوض بالقانون ونطلع $-\frac{\pi}{2}$

$$\left. + \frac{\pi}{2}\,\frac{\sin \pi k}{2k} - 0 + \frac{1}{2k}\,\frac{\cos 2kx}{2k}\Big|_{0}^{\frac{\pi}{2}} \right]$$

$\cos 0 = 1$

$$a_k = \frac{2}{\pi}\left[-\frac{1}{2k}\left(\frac{1}{2k} - \frac{\cos \pi k}{2k}\right) + \frac{1}{2k}\left(\frac{\cos \pi k}{2k} - \frac{1}{2k}\right) \right]$$

$$= \frac{2}{\pi(2k)^2}\left[-1 + \cos \pi k + \cos k\pi - 1\right]$$

لما شلنا وبسطنا طلعنا عامل مشترك الأقواس كلها

$$= \frac{1}{2\pi k^2}(-2 + 2\cos \pi k) = \frac{1}{\pi k^2}(-1 + \cos \pi k)$$

$$= \begin{cases} 0 & k \text{ زوجي} \\ -\frac{2}{\pi k^2} & k \text{ فردي} \end{cases}$$

$\cos 0 = 1$

$(1 - 1) = 0$

$\cos 1 = -1$

$(-1 - 1) = -2$